Millicharged neutrino with anomalous magnetic moment in rotating magnetized matter

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Abstract

We consider a millicharged neutrino with nonzero magnetic moment in the presence of rotating and magnetized background matter. The exact solution of the corresponding modified Dirac equation for the neutrino wave function is found. The neutrino energy spectrum is obtained and the effect of neutrino energy quantization is discussed in details. We introduce a new kind of spin operator which is a superposition of longitudinal and transverse polarizations operators for description of the neutrino spin properties in the considered background environment. Within the quasi-classical approach to the problem, radius of the neutrino orbits is derived and the effective "matter induced Lorentz force" is introduced. It is shown that in the considered environment, and also in matter with nonzero gradient of density, neutrino moves with acceleration. In this case a new type of the electromagnetic neutrino radiation (termed "light of millicharged neutrino", $LC\nu$) can be produced. The considered problem is of interest for astrophysical applications, in particular in connection with the recently reported hints of ultra-high energy neutrinos $E = 1 \div 10$ PeV observed by IceCube [1].

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1. Introduction

Neutrino electromagnetic properties are among the most intriguing and exciting issues of modern particle physics. Within the Standard Model neutrinos are massless and have "zero" electromagnetic properties. However, as it is well known in different extensions of the Standard Model a massive neutrino has non trivial electromagnetic properties (for a review of the neutrino electromagnetic properties see [2, 3]). That is why it is often claimed that neutrino electromagnetic properties open "a window to new physics" [4].

A neutrino magnetic moment, as expected in the easiest generalization of the Standard Model, is very small and proportional to the neutrino mass, $\mu_\nu \approx 3 \times 10^{-19}\mu_B(m_\nu/1 \text{ eV})$ with μ_B being the electron Bohr magneton. Much greater values are predicted in various other Standard Model generalizations (for details see [2, 3]).

It is usually believed that the neutrino has zero electric charge. This can be attributed to gauge invariance and anomaly cancellation constraints imposed in the Standard Model. However, if the neutrino has a mass, the statement that a neutrino electric charge is zero is not so evident as it meets the eye. In theoretical models with the absence of hypercharge quantization the electric charge also gets "dequantized" [5–7] and as a result neutrinos may become electrically millicharged particles. Charge dequantization for a massive Dirac neutrino may occur, for instance, in the Standard Model extensions with right-handed neutrinos ν_R and also in a wide class of models that contain an explicit $U(1)$ symmetry. It should be mentioned that only Dirac neutrino may have electric charge.

The most severe experimental constraints on the electric charge [8, 9] and the magnetic moment [10, 11] of the neutrino are given by

$$q_\nu \leq 10^{-21}e_0, \quad \mu_\nu \leq 2.9 \times 10^{-11}\mu_B, \tag{1}$$

where q_ν is a module of the neutrino millicharge and $\mu_B = e_0/(2m_e)$.

Due to obvious "smallness" of neutrino electromagnetic properties it is reasonable to expect that these properties manifest themselves most spectacular under the influence of extreme external conditions, namely in strong electromagnetic fields and dense background matter. These conditions can be easily found in astrophysics and cosmology settings.

In this letter we consider new possible phenomena of neutrino electromagnetic properties manifestation, in particular effects of nonzero magnetic

moment and electric milllicharge, in case of the particle motion in dense magnetized matter.

It is well known [12–15] that in case of particles motion in external high intensive electromagnetic fields the most adequate way to account for fields influence is based on the method of exact solutions of quantum equations for particle wave function. The same method of exact solutions can be used (see, for instance, [16]) to describe particle (neutrinos and electrons, in particular) motion in dense background matter. In connection with evaluation of quantum theory [17, 18] of spin light of neutrino in matter $SL\nu$ [19] the method of exact solutions was applied for describing neutrino quantum states in dense matter.

Most pronouncedly the power of the method of exact solutions is demonstrated in studies of different particle interactions in extreme conditions peculiar for astrophysical and cosmological settings [20–24]. Most recently exact solutions for a neutrino (and for an electron) in presence of matter and electromagnetic fields of different configurations had been evaluated in [25–27] (see also [16–18, 28]).

In this letter we consider for the first time a new form of the modified Dirac equation that describes the motion of an electrically millicharged neutrino with anomalous magnetic moment (or any charged fermion including an electron) under the influence of an external electromagnetic field and also accounting for weak interaction with the background matter. In the next section we discuss in details derivation of the corresponding exact solution for the particle wave function. The exact expression for the energy spectrum is also obtained. Note that in order to treat the particles spin properties we introduce a new type of spin operator.

In Section 3 we consider more specific background matter conditions and derive the exact solution for the millichaged neutrino wave function in case of magnetized and rotating matter. These particular external conditions are typical for the astrophysical environment that can be found, for instance, in neutron stars. An interesting new result is the neutrino energy spectrum quantization, the effect originated due to both electromagnetic and weak interactions of the neutrino with the background.

In Section 4 within the quasi-classical interpretation using the neutrino energy spectrum we introduce the “effective Lorentz force” that defines the neutrino motion. This new force is induced not only by the electromagnetic interaction but by the weak interactions with rotating matter as well. Matter induced “charge”, “electric” and “magnetic” fields are introduced.

In Section 5 a new possible mechanism of the electromagnetic neutrino radiation in nonuniform matter due to neutrino millicharge ("Light of (milli) Charged Neutrino", $LC\nu$) is discussed. In particular, it is shown that the $LC\nu$ can be emitted in the case of ultra-high energy millicharged neutrino moving in extreme astrophysical media with nonzero gradient of matter density. This phenomenon can occur in extremely high density matter mostly composed of neutrons, such as neutron stars. The influence of the electron matter component on the $LC\nu$ power of different neutrino flavors is also discussed.

2. Millicharged neutrino wave function in matter and magnetic field

Consider a millicharged neutrino with nonzero magnetic moment moving in dense matter composed of neutrons in the presence of external magnetic field. In the most general case the modified Dirac equation for a fermion wave function with nonzero anomalous magnetic moment exactly accounting for interaction with matter and electromagnetic field is

$$\left(\gamma_\mu(p^\mu + q_0 A^\mu) - \frac{1}{2}\gamma_\mu(c_l + \gamma_5)f^\mu - \frac{i}{2}\mu\sigma_{\mu\nu}F^{\mu\nu} - m\right)\Psi(x) = 0, \qquad (2)$$

where $F^{\mu\nu}$ is the electromagnetic field tensor and μ is the particle's anomalous magnetic moment. For definiteness we suppose that a fermion has a negative charge $q = -q_0$.

The matter effective potential $V_m = \frac{1}{2}\gamma_\mu(c_l + \gamma_5)f^\mu$ includes both the neutral and charged current interactions of the fermion with the background particles. The explicit form of f^μ depends on matter composition and the background particles densities, speeds and polarizations. For the case of unpolarized matter at rest one obtains

$$f^\mu = \frac{G_F n}{\sqrt{2}}(1, 0, 0, 0), \qquad (3)$$

where G_F is the Fermi constant. In equations (2) and (3) one should use charge, mass and magnetic moment of the considered fermion. For the electron neutrino $c_l = c_{\nu_e} = 1$ and $n = -n_n$, whereas for the electron $c_l = c_e = 1 - 4\sin^2\theta_W$ and $n = n_n$. Here n_n is neutron density.

Equation (2) can be written in the Hamiltonian form $i\frac{\partial}{\partial t}\Psi(x) = \hat{H}\Psi(x)$ with the Hamilton operator given by

$$\hat{H} = \gamma_0\boldsymbol{\gamma}(\hat{\mathbf{p}} + q_0\mathbf{A}(x)) + \gamma_0 m + \gamma_0\Sigma_3\mu B + \frac{G_F n}{2\sqrt{2}}(c_l + \gamma_5), \tag{4}$$

where $\hat{\mathbf{p}} = i(\partial_x, \partial_y, \partial_z)$ and the magnetic filed potential is $A^\mu = (0, -\frac{yB}{2}, \frac{xB}{2}, 0)$.

Using the standard representation of the γ-matrixes the Hamiltonian (4) can be re-written in the matrix form

$$\hat{H} = \begin{pmatrix} \widetilde{G}n + m + \mu B & 0 & \hat{P}_3 - \frac{Gn}{2} & \hat{P}_1 - i\hat{P}_2 \\ 0 & \widetilde{G}n + m - \mu B & \hat{P}_1 + i\hat{P}_2 & -\hat{P}_3 - \frac{Gn}{2} \\ \hat{P}_3 - \frac{Gn}{2} & \hat{P}_1 - i\hat{P}_2 & \widetilde{G}n - m - \mu B & 0 \\ \hat{P}_1 + i\hat{P}_2 & -\hat{P}_3 - \frac{Gn}{2} & 0 & \widetilde{G}n - m + \mu B \end{pmatrix}, \tag{5}$$

where $\hat{\mathbf{P}} = \hat{\mathbf{p}} + q_0\mathbf{A}(x)$ is the "extended" momentum operator, and we use the notations

$$G = \frac{G_F}{\sqrt{2}}, \quad \widetilde{G} = \frac{c_l}{2}G. \tag{6}$$

In the coordinate representation (the cylindrical coordinate system is used) the operators are given by the following expressions: $\hat{P}_1 \pm i\hat{P}_2 = -ie^{\pm i\varphi}(\frac{\partial}{\partial r} \pm \frac{i}{r}\frac{\partial}{\partial\varphi} \mp \frac{eB}{2}r)$, $\hat{P}_3 = -i\frac{\partial}{\partial z}$. The eigenvectors of the Hamiltonian (4), which satisfy the equation $\hat{H}\Psi(x) = p_0\Psi(x)$, can be written in the form

$$\Psi(x) = Ce^{-i(p_o t - p_3 z)} \begin{pmatrix} C_1\mathcal{L}_s^{l-1}(\frac{q_0 B}{2}r^2)e^{i(l-1)\varphi} \\ iC_2\mathcal{L}_s^{l}(\frac{q_0 B}{2}r^2)e^{il\varphi} \\ C_3\mathcal{L}_s^{l-1}(\frac{q_0 B}{2}r^2)e^{i(l-1)\varphi} \\ iC_4\mathcal{L}_s^{l}(\frac{q_0 B}{2}r^2)e^{il\varphi} \end{pmatrix}, \tag{7}$$

where $\mathcal{L}_s^l(\frac{q_0 B}{2}r^2)$ is a Laguerre function ($N = l + s = 0, 1, 2...$) and C is the normalization constant. The indeterminacy in the solution (7) due to the presence of undetermined arbitrary coefficients C_i ($i = 1, 2, 3, 4$) still remains because the particle spin properties up to this point have not been specified.

Using the Hamiltonian (5) and the solution of eq.(2) in the form given by

(7) we can get the following system of equations for the coefficients C_i

$$\begin{aligned}
&\Big((\widetilde{G}n - p_0) + (m + \mu B)\Big) C_1 + \left(p_3 - \frac{Gn}{2}\right) C_3 + \sqrt{2Nq_0B}C_4 = 0, \\
&\Big((\widetilde{G}n - p_0) + (m - \mu B)\Big) C_2 - \left(p_3 + \frac{Gn}{2}\right) C_4 + \sqrt{2Nq_0B}C_3 = 0, \\
&\Big((\widetilde{G}n - p_0) - (m + \mu B)\Big) C_3 + \left(p_3 - \frac{Gn}{2}\right) C_1 + \sqrt{2Nq_0B}C_2 = 0, \\
&\Big((\widetilde{G}n - p_0) - (m - \mu B)\Big) C_4 - \left(p_3 + \frac{Gn}{2}\right) C_2 + \sqrt{2Nq_0B}C_1 = 0.
\end{aligned} \tag{8}$$

To advance in evaluation of the exact solution of eq. (2) one should describe the particle spin properties and to construct the spin operator that commutes with the Hamiltonian. For the considered case of both the magnetic field and background matter presence this problem has never been studied before. To solve this problem we recall the known spin operators for the case of the particle motion in a pure magnetic field (without matter presence) and in matter (without a magnetic field) and also account for the fact that the new spin operator we are searching for should turn into one of the known spin operators in the corresponding limits of the magnetic field or background matter is "swiching off": $\mu B \to 0$ or $Gn \to 0$. Thus we arrive to the following new spin operator

$$\hat{S} = \sin\alpha\, \hat{S}_{tr} - \cos\alpha \hat{S}_{long}, \quad \sin\alpha = \frac{\frac{Gn}{2}}{\sqrt{\left(\frac{Gn}{2}\right)^2 + (\mu B)^2}}, \tag{9}$$

which is a weighted superposition of the operators of the longitudinal and transverse polarizations,

$$\hat{S}_{tr} = \frac{\mathbf{\Sigma}\hat{\mathbf{P}}}{m}, \quad \hat{S}_{long} = \Sigma_3 + \frac{i}{m}\begin{pmatrix} 0 & -\sigma_0 \\ \sigma_0 & 0 \end{pmatrix} [\boldsymbol{\sigma} \times \hat{\mathbf{P}}]_3, \tag{10}$$

where σ_μ are the Pauli matrixes. The value of angle α is fixed by the values of the effective matter density n and the strength of the magnetic field B.

It is easy to check that the introduced new spin operator (9) commutates with the Hamiltonian (4). Thus, these two operators have the same set of the eigenvectors: $\hat{S}\Psi(x) = S\Psi(x)$.

We obtain the system for the coefficients C_i which describes spin properties of the wave function (7):

$$\begin{aligned}
&(m\cos\alpha - p_3\sin\alpha + mS)C_1 + \sqrt{2Nq_0B}(C_4\cos\alpha - C_2\sin\alpha) = 0,\\
&(m\cos\alpha - p_3\sin\alpha + mS)C_3 - \sqrt{2Nq_0B}(C_2\cos\alpha + C_4\sin\alpha) = 0,\\
&(m\cos\alpha - p_3\sin\alpha - mS)C_2 + \sqrt{2Nq_0B}(C_3\cos\alpha + C_1\sin\alpha) = 0,\\
&(m\cos\alpha - p_3\sin\alpha - mS)C_4 - \sqrt{2Nq_0B}(C_1\cos\alpha - C_3\sin\alpha) = 0.
\end{aligned} \tag{11}$$

Demanding for non-trivial solution of this system we obtain the eigenvalues of the spin operator (9),

$$S = \zeta\frac{1}{m}\sqrt{(m\cos\alpha - p_3\sin\alpha)^2 + 2Nq_0B}, \quad \zeta = \pm 1. \tag{12}$$

After that we can solve the systems of equations (8) and (11) and get the energy spectrum

$$p_0 = \varepsilon\sqrt{p_3^2 + 2Nq_0B + m^2 + \left(\frac{Gn}{2}\right)^2 + (\mu B)^2 - 2mS\sqrt{\left(\frac{Gn}{2}\right)^2 + (\mu B)^2}} + \widetilde{G}n \tag{13}$$

and the exact expressions for the spin coefficients

$$\begin{aligned}
C_1 &= \frac{1}{2}\sqrt{1 - \frac{m\cos\alpha - p_3\sin\alpha}{mS}}\sqrt{1 + \sin(\alpha+\beta)},\\
C_2 &= \frac{1}{2}\delta_1\zeta\sqrt{1 + \frac{m\cos\alpha - p_3\sin\alpha}{mS}}\sqrt{1 + \sin(\alpha-\beta)},\\
C_3 &= \frac{1}{2}\delta_2\sqrt{1 - \frac{m\cos\alpha - p_3\sin\alpha}{mS}}\sqrt{1 - \sin(\alpha+\beta)},\\
C_4 &= \frac{1}{2}\delta_3\zeta\sqrt{1 + \frac{m\cos\alpha - p_3\sin\alpha}{mS}}\sqrt{1 - \sin(\alpha-\beta)}.
\end{aligned} \tag{14}$$

Here we use the notations $\delta_1 = -\mathrm{sgn}[\sin\alpha + \cos\beta]$, $\delta_2 = \mathrm{sgn}[\cos(\alpha+\beta)]$, $\delta_3 = \mathrm{sgn}[\cos\alpha + \sin\beta]$ and introduce a new angle β,

$$\cos\beta = \frac{m\sin\alpha + p_3\cos\alpha}{p_0 - \widetilde{G}n}. \tag{15}$$

It is easy to show that $\sin\beta$ introduced in (14) is given by

$$\sin\beta = \frac{\sqrt{\left(\frac{Gn}{2}\right)^2 + (\mu B)^2} - mS}{p_0 - \widetilde{G}n}. \tag{16}$$

Finally, the normalization constant of the wave function (7) is

$$C = \frac{1}{\sqrt{L}}\sqrt{\frac{q_0 B}{2\pi}}. \tag{17}$$

It is worth to stress that equations (7), (12), (13), (14) and (17) represent the exact solution of the modified Dirac equation (2) which describes an electrically charged fermion (millicharged neutrino) with anomalous magnetic moment motion in presence of magnetized matter.

3. Millicharged neutrino in rotating matter and magnetic field

Consider the case of the rotating media where the matter potential f^μ in (2) is given by

$$f^\mu = -Gn_n(1, -\epsilon y\omega, \epsilon x\omega, 0). \tag{18}$$

Here ω is an angular frequency, where $\epsilon = \pm 1$ that corresponds to parallel and antiparallel directions of two vectors $\boldsymbol{\omega}$ and $\mathbf{B}$. As we shall see below, these two cases are quite different and may have different consequences for astrophysical applications.

The Hamiltonian (4) in the case of rotating matter becomes

$$\hat{H} = \gamma_0\boldsymbol{\gamma}\hat{\mathbf{P}} + \gamma_0 m + \gamma_0\Sigma_3\mu B - \frac{Gn_n}{2}(1+\gamma_5)(1+\epsilon\gamma_0\gamma_1\omega y - \epsilon\gamma_0\gamma_2\omega x). \tag{19}$$

From the equation $\hat{H}\Psi(x) = p_0\Psi(x)$ we obtain for the wave function components the following equations

$$\begin{aligned}
(p_0 + Gn_n + p_3)\psi_1 - (\hat{P}_1 - i\hat{P}_2 + i\epsilon Gn_n\omega r e^{-i\varphi})\psi_2 &= (m+\mu B)\psi_3,\\
(p_0 + Gn_n - p_3)\psi_2 - (\hat{P}_1 + i\hat{P}_2 - i\epsilon Gn_n\omega r e^{i\varphi})\psi_1 &= (m-\mu B)\psi_4,\\
(p_0 + p_3)\psi_3 - (\hat{P}_1 - i\hat{P}_2)\psi_4 &= (m+\mu B)\psi_1,\\
(p_0 - p_3)\psi_4 - (\hat{P}_1 + i\hat{P}_2)\psi_3 &= (m-\mu B)\psi_2.
\end{aligned} \tag{20}$$

It is not a trivial task to solve the system (20) generally. The problem is reasonably simplified in the limit of small m and μB. In fact for the

relativistic neutrino these parts give a negligible contribution to the energy. Thereby two pairs of the neutrino wave function components ψ_i decouple one from another. In this case the neutrino wave function is represented as a sum of two neutrino chiral states $\Psi(x) = \Psi_L(x) + \Psi_R(x)$, where

$$\Psi_L \equiv \frac{1+\gamma_5}{2}\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_R \equiv \frac{1-\gamma_5}{2}\Psi = \begin{pmatrix} 0 \\ 0 \\ \psi_3 \\ \psi_4 \end{pmatrix}. \tag{21}$$

The second pair of equations (20) does not contain the matter term. Thus right-handed chiral neutrino state, Ψ_R, is attributed to the sterile neutrino. For the components ψ_3 and ψ_4 we obtain the solution in the following form

$$\begin{aligned} \psi_3 &= \frac{1}{2}\sqrt{\frac{q_\nu B}{2\pi L}}\sqrt{1-\frac{p_3}{p_0^R}}\mathcal{L}_s^{l-1}\left(\frac{q_\nu B}{2}r^2\right)e^{i(l-1)\varphi}, \\ \psi_4 &= \frac{i}{2}\sqrt{\frac{q_\nu B}{2\pi L}}\sqrt{1+\frac{p_3}{p_0^R}}\mathcal{L}_s^{l}\left(\frac{q_\nu B}{2}r^2\right)e^{il\varphi}, \end{aligned} \tag{22}$$

that, in fact, describes charged particle motion in external magnetic field in vacuum. In this case energy spectrum is quantized due to presence of magnetic field and has the form

$$p_0^R = \sqrt{p_3^2 + 2Nq_\nu B}, \tag{23}$$

where $N = 0, 1, 2...$ correspond to the Landau levels in the magnetic field.

The first pair of equations (20) corresponds to the active left-handed neutrino Ψ_L. For the components ψ_1 and ψ_2 we obtain the solution in the form

$$\begin{aligned} \psi_1 &= \frac{1}{2}\sqrt{\frac{|2Gn_n\omega - \epsilon q_\nu B|}{2\pi L}}\sqrt{1-\frac{p_3}{p_0^L + Gn_n}}\mathcal{L}_s^{l-1}\left(\frac{|2Gn_n\omega - \epsilon q_\nu B|}{2}r^2\right)e^{i(l-1)\varphi}, \\ \psi_2 &= \frac{i}{2}\sqrt{\frac{|2Gn_n\omega - \epsilon q_\nu B|}{2\pi L}}\sqrt{1+\frac{p_3}{p_0^L + Gn_n}}\mathcal{L}_s^{l}\left(\frac{|2Gn_n\omega - \epsilon q_\nu B|}{2}r^2\right)e^{il\varphi}. \end{aligned} \tag{24}$$

Here the energy spectrum is given by

$$p_0^L = \sqrt{p_3^2 + 2N|2Gn_n\omega - \epsilon q_\nu B|} - Gn_n. \tag{25}$$

The solution (24) and (25) describes a charged particle motion in external magnetic field and rotating matter. The energy spectrum (25) represents the modified Landau levels that combine two different types of interaction, both weak and electromagnetic. Note that in the electric charge vanishing limit the solution (24) reduces to the result that was previously obtained in [16, 26].

4. Quasi-classical treatment

It is also interesting to consider a more general case of a millicharged neutrino moving in an external environment when, in addition to the magnetic field, a constant electric field is presented. As it follows from (2) and (19), the neutrino energy spectrum in this case is

$$p_0 = \sqrt{p_3^2 + 2N|2Gn_n\omega - \epsilon q_\nu B| + m^2} - Gn_n - q\phi, \tag{26}$$

where ϕ is the scalar potential of the electric field. It is possible to introduce the generalized "effective Lorentz force" (see also [16]) that accounts for neutrino interaction with background matter and external electromagnetic field

$$\mathbf{F}_{eff} = q_{eff}\mathbf{E}_{eff} + q_{eff}\left[\boldsymbol{\beta} \times \mathbf{B}_{eff}\right]. \tag{27}$$

Here $\boldsymbol{\beta}$ is the neutrino speed and

$$\begin{aligned} q_{eff}\mathbf{E}_{eff} &= q_m\mathbf{E}_m + q\mathbf{E}, \\ q_{eff}\mathbf{B}_{eff} &= |q_m B_m + \epsilon q B|\mathbf{e}_z, \end{aligned} \tag{28}$$

where $q_m, \mathbf{B}_m, \mathbf{E}_m$ are the matter induced "charge", "electric" and "magnetic" fields correspondingly,

$$q_m = -G, \quad \mathbf{E}_m = -\boldsymbol{\nabla} n_n, \quad \mathbf{B}_m = 2n_n\boldsymbol{\omega}, \tag{29}$$

and $\mathbf{e}_z$ is a unit of the rotation axis. These relations directly follow from the exact form of the obtained energy spectrum (26) and are in agreement with the most general form of the "matter induced Lorentz force" [16]. Note that the "effective Lorentz force" (27) is generated by both weak and electromagnetic interactions. The electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields reproduce the ordinary electrodynamical Lorentz force and the "matter induced Lorentz force" is due to $\mathbf{E}_m$ and $\mathbf{B}_m$.

As it follows from (27), (28) and (29), there are two types of forces acting on the millicharged neutrino moving in background matter with nonvanishing gradient of density in presence of external electric and magnetic fields. The first type is an accelerating force which is produced by the effective electric field $\mathbf{E}_{eff}$. The second one is the centripetal force which is produced by the effective magnetic field $\mathbf{B}_{eff}$. As it follows from (27), the second force is orthogonal to the neutrino speed $\boldsymbol{\beta}$. Note that possibility of particle acceleration due to the gradient of matter density was also discussed in [16, 29, 30].

Consider the motion of an active left-handed neutrino that occupies one of the excited modified Landau levels characterized by $N \gg 1$ in the case when the "effective electric field" $\mathbf{E}_{eff}$ (28) is not presented. This situation may happen, for instance, for a neutrino moving in a rotating neutron star in the transversal to the rotating axis plane. In this case the radius of the neutrino quasi-classical orbit

$$R = \int_0^\infty \Psi_L^\dagger \, \mathrm{r} \, \Psi_L \, d\mathbf{r} \tag{30}$$

depends on the parameter ϵ and has the following form

$$R = \sqrt{\frac{2N}{|2Gn_n\omega - \epsilon q_\nu B|}}. \tag{31}$$

It follows that for the fixed values of the magnetic field strength $\mathbf{B}$ and the rotation angular frequency $\boldsymbol{\omega}$ the radius of the neutrino orbit gets different values for two different values of ϵ. When the angular frequency $\boldsymbol{\omega}$ is antiparallel to the magnetic field $\mathbf{B}$ ($\epsilon = -1$) the radius (31) decreases when both the angular frequency and magnetic field strength increase. Note that neutrinos can have bound orbits inside a rotating neutron star. The orbit radii of neutrino with quantum number N up to 10^{10} and with energy up to 1 eV are less than the radius of neutron star. Thus low energy neutrinos can be bounded inside a rotating neutron star.

In the opposite case when two vectors $\boldsymbol{\omega}$ and $\mathbf{B}$ are parallel ($\epsilon = +1$), the effect of weak interactions with rotating matter and the millicharge interaction with magnetic field act in opposite directions and under the condition $2Gn_n\omega = q_\nu B$ can even "kill" each other. In this case the radius tends to infinity, i.e. neutrino propagates along a straight line as in a free motion. Note that this condition can be realized for reasonable choice of parameters of a neutron star ($n_n \sim 10^{37} cm^{-3}$, $\omega = 2\pi \times 10^3 s^{-1}$, and $B \sim 10^{12} G$).

5. Light of millicharged neutrino ($LC\nu$)

In more general case of rotating matter with nonconstant density and in presence of electric and magnetic fields a massive neutrino is moving with acceleration under the action of the effective Lorentz force (28)

$$\mathbf{a} = (G\boldsymbol{\nabla} n_n + q_\nu \boldsymbol{\nabla}\phi + |2Gn_n\omega - \epsilon q_\nu B|\,[\boldsymbol{\beta}\times\mathbf{e}_z])\,\frac{1}{m}. \tag{32}$$

The corresponding total power of the electromagnetic radiation due to neutrino millicharge (in the quasi-classical limit) is given by

$$I_{LC\nu} = \frac{2q_\nu^2}{3}\left(\frac{\mathbf{a}^2}{(1-\boldsymbol{\beta}^2)^2} + \frac{(\mathbf{a}\boldsymbol{\beta})^2}{(1-\boldsymbol{\beta}^2)^3}\right). \tag{33}$$

We tern the considered mechanism of the neutrino electromagnetic radiation due to neutrino millicharge, that can be emitted in the presence of background matter and electromagnetic fields, as the "Light of (milli)Charged Neutrino" ($LC\nu$).

Consider possible applications for the obtained results in different astrophysical environments and, in particular, in neutron stars. A neutron star is a very compact astrophysical object ($R_{NS} \sim 10\,km$) with a very dense neutron core and a crust around it. The crust comprises a relatively small fraction of total radius of the star (usually about 10%). Radial density distributions for different theoretical models of neutron stars had been considered in [31]. In particular, core density is extremely high (near the nuclei density $\rho_n \sim 10^{14} g/cm^3$) and is almost uniform. In turn, crust density rapidly decreases from ρ_n at the bottom to zero at the star surface.

In case when the massive neutrino has both nonzero electric millicharge and magnetic moment it is interesting to compare the efficiency of the proposed new mechanizm of the electromagnetic radiation by a millicharged neutrino $LC\nu$ with the spin light of neutrino in matter ($SL\nu$) [17, 18]. The $SL\nu$ can be emitted due to the neutrino magnetic moment and was first proposed and studied within the quasi-classical treatment in [19]. For neutrino electromagnetic characteristics we use the values that are given by the present experimental limits (1).

We consider the radiation processes, $LC\nu$ and $SL\nu$, during propagation of the neutrino from the central part of the star outwards for two specific regions. In the neutron core the $SL\nu$ radiation power, given by

$$I_{SL\nu} = \frac{4}{3}\mu^2\alpha_m^2 m^2 E^2, \quad \alpha_m = -\frac{1}{m}\frac{Gn_n}{2}, \tag{34}$$

dominates over the power of the $LC\nu$. However, in the crust of the neutron star the rapid decrease of density is crucial. Therefore the first term of equation (32) gives a leading contribution to the $LC\nu$. From (33) for the $LC\nu$ radiation power in the crust of a neutron star we obtain

$$I_{LC\nu} = \frac{2}{3} q_\nu^2 (\boldsymbol{\nabla} \alpha_m)^2 \gamma^6. \tag{35}$$

Thus the ratio of radiation powers $I_{LC\nu}$ and $I_{SL\nu}$ for the neutrino mass $m = 0.05\, eV$ reads

$$\frac{I_{LC\nu}}{I_{SL\nu}} \simeq \left(\frac{E^2}{10\, eV^3} \frac{\boldsymbol{\nabla} n_n}{n_n} \right)^2. \tag{36}$$

For the average density gradient value $|G \boldsymbol{\nabla} n_n| \sim 1\, eV/1\, km$ and the neutrino energy $E = 10\, MeV$ we obtain

$$\frac{I_{LC\nu}}{I_{SL\nu}} \simeq \left(\frac{10^5\, eV}{G n_n} \right)^2, \tag{37}$$

The maximum value $\sim 1\, eV$ for the quantity $G n_n$ can be found in the central core and this value decreases to the surface.

From the estimation (37) it just straightforward that in case of millicharged neutrino with nonzero magnetic moment the mechanism of electromagnetic radiation due to neutrino charge ($LC\nu$) is more effective than the electromagnetic radiation due to the neutrino magnetic moment ($SL\nu$) in dense matter, peculiar for the crust of a neutron star.

Note that different flavor neutrinos exhibit in general different behavior in respect to the introduced new $LC\nu$ mechanism. In fact, in the neutron star there is a small fraction of electrons and protons (about 10%) in addition to neutrons. Due to electroneutrality of matter the concentrations of electrons and protons are equal, $n_e = n_p$. In this case the $LC\nu$ for the electron neutrino has the form

$$I_{LC\nu_e} = \frac{2}{3} \frac{q_\nu^2}{m^2} \gamma^6 \left(G \boldsymbol{\nabla} (2n_e - n_n) \right)^2. \tag{38}$$

We expect that the direct Coulomb interaction of an extremely small neutrino millicharge with charged particles of matter is negligible.

However, weak interactions of neutrino with matter with complex composition depend on the neutrino flavor. And for the $LC\nu$ for muon and tau neutrinos we obtain again (35). Thus we predict different rates of $LC\nu$ for

different neutrino flavors. The ratio of radiation powers of neutrinos with the same energies is given by

$$\frac{I_{LC\nu_e}}{I_{LC\nu_{\mu,\tau}}} = \frac{(\boldsymbol{\nabla} n_n - 2\boldsymbol{\nabla} n_e)^2}{(\boldsymbol{\nabla} n_n)^2}. \tag{39}$$

Therefore nonuniform electron component of electrically neutral matter changes the radiation power of the $LC\nu_e$.

6. Conclusions

We consider electromagnetic properties of a neutrino with an electric millicharge and anomalous magnetic moment moving in an external magnetic field and dense matter on the basis of the exact solutions of the modified Dirac equation for the particle wave function in the external environment. Possible effects of matter rotation and effects of nonzero matter gradient are also included. To solve the modified Dirac equation in this background environment we introduce a new type of spin operator which is a superposition of the operators of the longitudinal and transverse polarizations. The spectra of the proposed spin operator and particle's energy are obtained. These exact solutions can be applied not only to neutrinos but to other fermions moving in the considered background.

From the obtained exact solutions it follows that the neutrino energy spectrum is quantized due to both electromagnetic interaction of the neutrino millicharge with external magnetic field and weak interaction of the neutrino with the rotating background particles. These two phenomena overlap and, depending on the orientation of matter rotation axis and the direction of magnetic field, the effect of energy quantization can be reinforced or washed out.

In the quasi-classical treatment, the energy spectrum obtained for the magnetized rotating matter corresponds to the neutrino motion on circular orbits very much similar to the Landau orbits of an electron in the external magnetic field. Considering background conditions peculiar for a dense rotating neutron star we predict that low energy neutrinos can be bound inside the star.

We also introduce the generalized effective Lorentz force that accounts for the millicharged neutrino interaction with external magnetic field and rotating background matter with nonzero gradient of density. In general,

under the influence of the effective Lorentz force neutrino can be accelerated. This new possible mechanism of the electromagnetic radiation by the millicharged neutrino moving in magnetized nonuniform rotating matter has been introduced and termed the "Light of (milli)Charged Neutrino" ($LC\nu$). The obtained radiation power of the $LC\nu$ exhibits strong dependence of the neutrino γ-factor (to the six order) and also on the gradient of matter density. It is expected that this mechanism can be efficient in different astrophysical environments with rather rapid decrease (increase) of matter density.

The considered new manifestations of nontrivial neutrino electromagnetic properties, due to their strong dependence on the neutrino energy, are of interest for astrophysical applications, in particular in connection with the recently reported hints of ultra-high energy PeV neutrinos observed by IceCube [1].

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